1.

a) $\lim\limits_{x\to+\infty}(3x^4+2x^3+1) = \lim\limits_{x\to+\infty} \underset{\downarrow\, +\infty}{x^4}\,(\underset{\downarrow\, 3}{3}+\underset{\downarrow\, 0}{\frac{2}{x}}+\underset{\downarrow\, 0}{\frac{1}{x^4}}) = [+\infty\cdot 3] = +\infty$

b) $\lim\limits_{x\to-\infty}\frac{4x^6+5x^5+2}{x^3+x-1} = \lim\limits_{x\to-\infty}\frac{\overset{-\infty \Leftarrow\, x^3}{\cancel{x^6}}(\overset{\Rightarrow 4}{4}+\overset{\Rightarrow 0}{\frac{5}{x}}+\overset{\Rightarrow 0}{\frac{2}{x^6}})}{\underset{1}{\cancel{x^3}}(\underset{\downarrow\, 1}{1}+\underset{\downarrow\, 0}{\frac{1}{x^2}}-\underset{\downarrow\, 0}{\frac{1}{x^3}})} = \left[\frac{-\infty\cdot 4}{1}\right] = -\infty$

c) $\lim\limits_{x\to-\infty}\left(\sqrt{3-2x}-\sqrt{4-2x}\right) = \lim\limits_{x\to-\infty}\frac{(\sqrt{3-2x}-\sqrt{4-2x})(\sqrt{3-2x}+\sqrt{4-2x})}{\sqrt{3-2x}+\sqrt{4-2x}} =$

$= \lim\limits_{x\to-\infty}\frac{3-\cancel{2x}-4+\cancel{2x}}{\sqrt{3-2x}+\sqrt{4-2x}} = \lim\limits_{x\to-\infty}\frac{-1}{\sqrt{3-2x}+\sqrt{4-2x}} = \left[\frac{-1}{+\infty}\right] = 0$

d) $\lim\limits_{x\to-3}\frac{4-x}{(x+4)^8} = \left[\frac{4+3}{(-3+4)^8}\right] = \left[\frac{7}{1}\right] = 7$

e) $\lim\limits_{x\to4}\frac{3x^2-11x-4}{x^2-3x-4} = \left[\frac{0}{0}\right] = \lim\limits_{x\to4}\frac{3(x+\frac{1}{3})\cancel{(x-4)}}{(x+1)\cancel{(x-4)}} = \left[\frac{3(4+\frac{1}{3})}{4+1}\right] = \frac{13}{5}$

$x^2-3x-4=0$	$3x^2-11x-4=0$
$\Delta=25$	$\Delta=169$
$x_1=-1$	$x_1=-\frac{1}{3}$
$x_2=4$	$x_2=4$

f) $\lim\limits_{x\to-\infty}\frac{4x\sqrt{3x^2+19}}{3x^2+7} = \boxed{\frac{\infty}{\infty}} = \lim\limits_{x\to-\infty}\frac{4x\sqrt{x^2(3+\frac{19}{x^2})}}{x^2(3+\frac{7}{x^2})} = \lim\limits_{x\to-\infty}\frac{4x|x|\sqrt{3+\frac{19}{x^2}}}{x^2(3+\frac{7}{x^2})}$

$= \lim\limits_{x\to-\infty}\frac{-4\cancel{x^2}\overset{\sqrt{3}}{\sqrt{3+\overset{\Rightarrow 0}{\frac{19}{x^2}}}}}{\cancel{x^2}(\underset{\downarrow\, 3}{3}+\underset{\downarrow\, 0}{\frac{7}{x^2}})} = \left[\frac{-4\cdot\sqrt{3}}{3}\right] = \frac{-4\sqrt{3}}{3}$

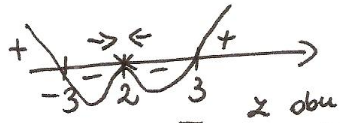
g) $\lim\limits_{x\to+\infty}\sqrt{3x^2+8} = +\infty$

h) $\lim\limits_{x\to 2} \frac{3x-12}{(x^2-9)(x-2)^2} = \left[\frac{-6}{0^-}\right] = +\infty$

$(x^2-9)(x-2)^2=0$

$x^2=9 \qquad x=2 \leftarrow$ 2krotny

$x=3 \lor x=-3$



z obu stron $x=2$ ten sam znak $(-)$.

i) $\lim\limits_{x\to +\infty} \frac{5x^{10}+2x}{-x^{10}+x^8+2x^4} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to +\infty} \frac{\cancel{x^{10}}(5+\overbrace{\frac{2}{x^9}}^{\to 0})}{\cancel{x^{10}}(-1+\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{2}{x^6}}_{\to 0})} = \left[\frac{5}{-1}\right] = -5$

j) $\lim\limits_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 2} \frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{(x-2)(\sqrt{x+2}+2)} =$

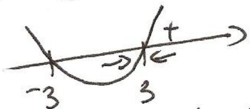
$= \lim\limits_{x\to 2} \frac{x+2-4}{(x-2)(\sqrt{x+2}+2)} = \lim\limits_{x\to 2} \frac{\cancel{x-2}\ 1}{\cancel{(x-2)}(\sqrt{x+2}+2)} = \left[\frac{1}{\sqrt{2+2}+2}\right] = \frac{1}{4}$

k) $\lim\limits_{x\to 0} \frac{x^3-5x}{x^4+2x^3+3x^2+x} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 0} \frac{\cancel{x}(x^2-5)}{\cancel{x}(x^3+2x^2+3x+1)} = \left[\frac{-5}{1}\right] = -5$

l) $\lim\limits_{x\to 3} \frac{4x-1}{x^2-9} = \left[\frac{11}{0}\right] = \pm\infty \leftarrow$ granica nie istnieje

$x^2-9=0$

$x=3 \lor x=-3$



Po obu stronach $x=3$, znaki są różne $(+,-)$

m) $\lim\limits_{x\to \infty} \frac{2}{\sqrt{x}-\sqrt{x+3}} = \left[\frac{2}{\infty-\infty}\right] = \lim\limits_{x\to\infty} \frac{2(\sqrt{x}+\sqrt{x+3})}{(\sqrt{x}-\sqrt{x+3})(\sqrt{x}+\sqrt{x+3})} =$

$= \lim\limits_{x\to\infty} \frac{2(\sqrt{x}+\sqrt{x+3})}{\cancel{x}-\cancel{x}-3} = \lim\limits_{x\to\infty} -\frac{2}{3}(\sqrt{x}+\sqrt{x+3}) = \left[-\frac{2}{3}(\infty+\infty)\right] = -\infty$

n) $\lim\limits_{x\to -\infty} \frac{x(3x+2)(4-x)}{(2x-1)^2} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to -\infty} \frac{x\cdot x(3+\frac{2}{x})\cdot x(\frac{4}{x}-1)}{(x(2-\frac{1}{x}))^2} =$

$= \lim\limits_{x\to -\infty} \frac{\cancel{x^3}(3+\frac{2}{x})\cdot(\frac{4}{x}-1)}{\cancel{x^2}(2-\underbrace{\frac{1}{x}}_{\to 0})^2} = \left[\frac{\infty\cdot 3\cdot(-1)}{2}\right] = -\infty$

o) $\lim\limits_{x\to -3^+} \frac{x^3+2}{x^2+x-6} = \left[\frac{-25}{0^-}\right] = +\infty$

$x^2+x-6=0$
$\Delta=25$
$x_1=-3$
$x_2=2$

p) $\lim\limits_{x\to+\infty}(-3x^5-x^2+1)(5-x^3-2x) = [-\infty\cdot -\infty] = \lim\limits_{x\to+\infty} x^5\left(-3-\frac{1}{x^3}+\frac{1}{x^5}\right)\cdot x^3\left(\frac{5}{x^3}-1-\frac{2}{x^2}\right) =$

$=\lim\limits_{x\to+\infty} \underset{+\infty}{\underset{\downarrow}{x^8}}\left(\underset{-3}{\underset{\downarrow}{-3}}-\underset{0}{\underset{\downarrow}{\frac{1}{x^3}}}+\underset{0}{\underset{\downarrow}{\frac{1}{x^5}}}\right)\left(\underset{0}{\underset{\downarrow}{\frac{5}{x^3}}}\underset{-1}{\underset{\downarrow}{-1}}-\underset{0}{\underset{\downarrow}{\frac{2}{x^2}}}\right) = [+\infty\cdot(-3)\cdot(-1)] = +\infty$

r) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-x\right) = [\infty-\infty] = \lim\limits_{x\to+\infty}\frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x} =$

$=\lim\limits_{x\to+\infty}\frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}+x} = \left[\frac{1}{\infty}\right] = 0$

s) $\lim\limits_{x\to -1^-}\frac{x^2}{x^2-1} = \left[\frac{1}{0^+}\right] = +\infty$

$x^2-1=0$
$x=1 \lor x=-1$

t) $\lim\limits_{x\to 1}\frac{x^2-2x+1}{x^2-1} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 1}\frac{(x-1)^{\cancel{2}}}{\cancel{(x-1)}(x+1)} = \lim\limits_{x\to 1}\frac{x-1}{x+1} = \left[\frac{0}{2}\right] = 0$

u) $\lim\limits_{x\to+\infty}\frac{3x\sqrt{x^2+7}}{5x^2+13} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to+\infty}\frac{3x\sqrt{x^2(1+\frac{7}{x^2})}}{x^2(5+\frac{13}{x^2})} = \lim\limits_{x\to+\infty}\frac{3x\cdot|x|\sqrt{1+\frac{7}{x^2}}}{x^2(5+\frac{13}{x^2})} =$

$=\lim\limits_{x\to+\infty}\frac{3\cancel{x^2}\sqrt{1+\frac{7}{x^2}}}{\cancel{x^2}(5+\frac{13}{x^2})} = \left[\frac{3\cdot\sqrt{1}}{5}\right] = \frac{3}{5}$

w) $\lim\limits_{x\to 4}\frac{\sqrt{1+2x}-\sqrt{x+5}}{16-x^2} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 4}\frac{(\sqrt{1+2x}-\sqrt{x+5})(\sqrt{1+2x}+\sqrt{x+5})}{(16-x^2)(\sqrt{1+2x}+\sqrt{x+5})} =$

$=\lim\limits_{x\to 4}\frac{1+2x-x-5}{(16-x^2)(\sqrt{1+2x}+\sqrt{x+5})} = \lim\limits_{x\to 4}\frac{\cancel{x-4}^{\,1}}{-\cancel{(x-4)}_{1}(x+4)(\sqrt{1+2x}+\sqrt{x+5})} =$

$=\left[\frac{1}{-1\cdot 8\cdot(\sqrt{9}+\sqrt{9})}\right] = \left[\frac{1}{-8\cdot 6}\right] = -\frac{1}{48}$

5.

a) $\lim\limits_{x \to 0^+} (x+1) = 1$

$\lim\limits_{x \to 0^-} (-x+1) = 1$

$\Rightarrow \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} f(x) = \underline{1}$

d) $\lim\limits_{x \to 2^+} x = 2$

$\lim\limits_{x \to 2^-} \left(-\frac{1}{x^2}\right) = -\frac{1}{4}$

$\Rightarrow \lim\limits_{x \to 2^+} f(x) \neq \lim\limits_{x \to 2^-} f(x)$ więc

$\lim\limits_{x \to 2} f(x)$ nie istnieje

e) $\lim\limits_{x \to 0^+} \frac{x^2}{|x|} = \left[\frac{0}{0}\right] = \lim\limits_{x \to 0^+} \frac{x^2}{x} = \lim\limits_{x \to 0^+} x = 0$

$\lim\limits_{x \to 0^-} \frac{x^2}{|x|} = \lim\limits_{x \to 0} \frac{x^2}{-x} = \lim\limits_{x \to 0} (-x) = 0$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} f(x) = 0$